

Linear Motion and Force

Problem Set 18: Energy, Work and Power

18.1
$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(42\text{kg})(3.7\text{ms}^{-1})^2 = 287\text{ J}$$

18.2
$$E_p = mgh = (1000\text{kg})(9.8\text{ms}^{-2})(3.0\text{m}) = 29,400\text{ J (or 29.4 kJ)}$$

18.3 [a]
$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(0.5\text{kg})(8\text{ms}^{-1})^2 = 16\text{ J}$$

[b] Gain in KE = loss in PE = $mgh = (0.5\text{kg})(9.8\text{ms}^{-2})(45.0\text{m}) = 221\text{ J}$
New KE = 16 J + 221 J = 237 J

18.4 [a]
$$W = Pt = (25\text{W})(24\text{h})(3600\text{ s h}^{-1}) = 2.16 \times 10^6\text{ J (or 2.16 MJ)}$$

[b] Not all the electrical energy supplied by the pump will be transferred usefully – some will be lost as heat and sound energy.

18.5 [a]
$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(2000\text{kg})(18\text{ms}^{-1})^2 = 3.24 \times 10^5\text{ J (or 324 kJ)}$$

[b]
$$F = \frac{W}{s} = \frac{3.24 \times 10^5\text{ J}}{48\text{m}} = 6750\text{ N (in the opposite direction to the van's original motion)}$$

18.6 [a]
$$W = Fs = (8\text{N})(2\text{m}) = 16\text{ J}$$

[b]
$$W = Fs = (12\text{N})(0.55\text{m}) = 6.6\text{ J}$$

[c]
$$W = Fs = (25\text{N})(1.8\text{m}) = 45\text{ J}$$

[d]
$$W = Fs = (1015\text{kg})(9.8\text{ms}^{-2})(310\text{m}) = 3.08 \times 10^6\text{ J (or 3.08 MJ)}$$

[e] Since $a = \text{zero}$, $F = F_{\text{friction}}$, then $W = Fs = (42\text{N})(10\text{m}) = 420\text{ J}$

18.7
$$v^2 = u^2 + 2gs$$

$$E_k = \frac{1}{2}mv^2$$

$$\text{So, } E_k = \frac{1}{2}m(u^2 + 2gs) = \frac{1}{2}(0.2\text{kg})((10\text{ms}^{-1})^2 + (2)(9.8\text{ms}^{-2})(12\text{m})) = 33.5\text{ J}$$

18.8 [a]
$$W = Fs = (12\text{N})(18\text{m}) = 216\text{ J}$$

[b]
$$F = \frac{W}{s} = \frac{100\text{ J}}{0.8\text{m}} = 125\text{ N upwards}$$

[c]
$$s = \frac{W}{F} = \frac{1200\text{ J}}{30\text{ N}} = 40.0\text{ m}$$

18.9
$$! = \frac{W}{F} = \frac{100\text{ J}}{125\text{ N}} = 0.80\text{ m}$$

Linear Motion and Force

$$18.10 \quad F_{av} = \frac{W}{s} = \frac{3000J}{25m} = 120 N$$

$$18.11 \quad W = Fs = (2000kg)(9.8ms^{-2})(50m) = 9.8 \times 10^5 J \text{ (or } 980 \text{ kJ)}$$

$$18.12 \text{ [a]} \quad P = \frac{W}{t} = \frac{5000J}{12s} = 417 W$$

$$\text{[b]} \quad P = \frac{W}{t} = \frac{mgh}{t} = \frac{(2.2kg)(9.8ms^{-2})(2.3m)}{0.8s} = 62.0 W$$

$$\text{[c]} \quad E_k = \frac{1}{2}mv^2 = \frac{1}{2}(0.355kg)(20ms^{-1})^2 = 71 J$$

$$P = \frac{W}{t} = \frac{71J}{1.2s} = 59.2 W$$

$$\text{[d]} \quad P = \frac{W}{t} = \frac{Fs}{t} = \frac{(35N)(15m)}{3.5s} = 150 W$$

18.13 [a] On contact with the soft sand the athlete's feet take longer to come to rest and they move further into the surface so they have to work harder to raise them and get them moving again. So they use more of the chemical energy they have acquired from their food and their muscles have to expend more of their stored elastic potential energy. In one hour, their body would generate more power.

[b] Raising your centre of gravity means hurdlers have to use some of their chemical and elastic potential energy to overcome the gravitational force, meaning they have less energy to transfer into kinetic energy and therefore will not run as quickly.

[c] Roller skates effectively make contact with the ground at a point rather than over a larger surface area which is the situation with the soles of your feet, therefore they work against a much smaller frictional force. This means that your legs can apply a much smaller push force to move, using less energy.

[d] Crouching offers a much smaller surface area therefore a reduced drag or air resistance force at the start of the sprint. This means that more of the sprinter's chemical and elastic potential energy can be used to generate a greater initial acceleration burst, producing an increase in speed more quickly.

[e] When walking up a hill, you have to overcome an additional gravitational force, meaning you have less energy to transfer into kinetic energy and therefore you will have to work harder to cover the same distance.

[f] The more kinetic energy a long jumper has on take off the more gravitational potential energy they will acquire while in the air. This means they should be able to gain additional height and stay in the air longer, thereby producing a longer jump.

Linear Motion and Force

- [g] The Fosbury Flop technique does not involve the high jumper's centre of gravity rising as high above the ground as the Scissor Kick technique. See part [b] for further explanation
- [h] When high jumpers push their arms forward, they are using Newton's third law (equal and opposite reaction) to gain an additional push from the ground in order that they jump with a greater force.
- 18.14 [a] Since $F = \frac{m(v-u)}{t}$: in a pile driver, a large mass m is (almost) instantly brought to rest. Hence the time, t is very short; so the force of impact F on the pile is huge.
- [b] To make the pile driver more effective you could:
- Use a heavier weight (increase m)
 - Create a greater swing before the makes contact with the pile (increase ΔV)
 - Have the lower end of the pile shaped like a point, the smaller surface area thereby creating greater pressure as it drives into the ground
- 18.15 [a] Loss in $E_k = \text{gain in } E_p$
 $E_k = \frac{1}{2}mv^2 = mgh$ (note that the mass, m is the same on both sides and thus cancels)
 $v_{min} = \sqrt{2gh} = \sqrt{2(9.8ms^{-2})(10m)} = 14.0 ms^{-1}$
- [b] The initial E_k has to overcome a frictional force as it climbs up the slope and so it will lose some energy as heat and sound energy. So it needs a greater amount of E_k to begin with in order that it can acquire the necessary E_p to reach the top of the slope.
- 18.16 [a] $P = \frac{W}{t} = \frac{Fs}{t} = F \times (\text{gradient of graph}) = (256kg \times 9.8ms^{-2}) \times \frac{14m}{8s} = 4390 W$
- [b] $P = \frac{W}{t} = \frac{Fs}{t} = F \times v_{av}$
 Speed of hoist $v_{av} = \frac{P}{F} = \frac{4390W}{(2750kg)(9.8ms^{-2})} = 0.163 ms^{-1}$
- 18.17 Initially, $E_k = \frac{1}{2}mv^2 = \frac{1}{2}(3.6kg)(130ms^{-1})^2 = 3.04 \times 10^4 J$ (or 30.4 kJ)
 Finally, $E_k = \frac{1}{2}mv^2 = \frac{1}{2}(3.6kg)(65ms^{-1})^2 = 7.61 \times 10^3 J$ (or 7.61 kJ)
 $W = \text{loss in KE} = 30.4 kJ - 7.61 kJ = 22.8 kJ$

Linear Motion and Force

18.18 [a] Theoretical gain in $E_k = \text{loss in } E_p = mgh = (64\text{kg})(9.8\text{ms}^{-2})(250\text{m}) = 157,000\text{J}$
(or 157 kJ)

[b] Real gain in $E_k = 20\%$ of $157\text{kJ} = 31.4\text{ kJ}$

$$[c] \quad v = \sqrt{\frac{2E_k}{m}} = \sqrt{2 \times \frac{31400\text{J}}{64\text{kg}}} = 31.3\text{ ms}^{-1}$$

18.19 [a] After 2s, $E_k = 600\text{ J}$

$$P = \frac{W}{t} = \frac{600\text{J}}{2\text{s}} = 300\text{ W}$$

$$[b] \quad P = \frac{W}{t} = \frac{3000 - 1500\text{J}}{20\text{s} - 10\text{s}} = 150\text{ W}$$

$$[c] \quad P = \frac{W}{t} = \frac{4000\text{J} - 3000\text{J}}{40\text{s} - 25\text{s}} = 66.7\text{ W}$$

[d] Chris is changing gears on his bike so there is no increase in speed, therefore no increase in E_k .

18.20 Change in $E_p = mgh = (2.5 \times 10^{11}\text{kg})(9.8\text{ms}^{-2})(1.75\text{m}) = 4.29 \times 10^{12}\text{ J}$

The station can only transfer 20% of this energy usefully,

so energy supplied = $8.58 \times 10^{11}\text{ J}$

18.21 [a] Vertical height, $h = s \times \sin\theta = 12\text{m} \times \sin 15^\circ = 3.11\text{m}$

$$E_p = mgh = (5\text{kg})(9.8\text{ms}^{-2})(3.11\text{m}) = 152\text{ J}$$

$$[b] \quad E_k = \frac{1}{2}mv^2 = \frac{1}{2}(5\text{kg})(6\text{ms}^{-1})^2 = 90.0\text{ J}$$

$$[c] \quad W_{\text{friction}} = \text{loss in energy} = 152\text{ J} - 90\text{ J} = 62.2\text{ J}$$

$$[d] \quad F_{\text{av}} = \frac{W_{\text{friction}}}{s} = \frac{62.2\text{J}}{12\text{m}} = 5.18\text{ N up the ramp}$$

18.22 [a] $F_{\text{friction}} = (0.1)(213)(9.8\text{ms}^{-2}) = 209\text{ N}$

$$W_{\text{friction}} = FS = 209\text{N} \times 25.3\text{m} = 5290\text{ J}$$

Vertical height moved through, $h = s \times \sin\theta = 25.3\text{m} \times \sin 30^\circ = 12.7\text{m}$

$E_p = mgh = (213\text{kg})(9.8\text{ms}^{-2})(12.7\text{m}) = 26,400\text{ J}$ (this is the work done overcoming gravity)

So the bike must expend a total energy = $5290\text{J} + 26400\text{J} = 31,700\text{ J}$ (or 31.7 kJ)

$$[b] \quad \text{Time up slope: } t = \frac{d}{v_{\text{av}}} = \frac{25.3\text{m}}{12\text{ms}^{-1}} = 2.11\text{ s}$$

$$P = \frac{W}{t} = \frac{31700\text{J}}{2.11\text{s}} = 15035\text{ W (or 15.1 kW)}$$

18.23 $W = Fscos\theta = (20\text{N})(54\text{m})\cos 45^\circ = 764\text{ J}$